Factorials and Circular Permutations

take note

Key Concept *n* Factorial

For any positive integer n, n factorial is $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$ and is written as $n!$. Zero factorial, or $0!$, is defined to be 1.

Example $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

Factorial Examples:

1) $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

2) $\frac{10!}{5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30{,}240$

3) $5! \times 6! = (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) \cdot (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 86{,}400$

4) You download 8 songs on your phone. If you play the songs using the random shuffle option, how many different ways can the sequence of songs be played?

$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$

KeyConcept Circular Permutations

The number of distinguishable permutations of n objects arranged in a circle with no fixed reference point is

$$\frac{n!}{n} \text{ or } (n-1)!.$$

If the n objects are arranged relative to a fixed reference point, then the arrangements are treated as linear, making the number of permutations $n!$.

JEWELRY **If the 6 charms on the bracelet shown are arranged at random, what is the probability that the arrangement shown is produced?**

Since there is no fixed reference point, this is a circular permutation. So, there are $(6-1)!$ or $5!$ distinguishable permutations of the charms. Thus, the probability that the exact arrangement shown is produced is $\frac{1}{5!}$ or $\frac{1}{120}$.

DINING **You are seating a party of 4 people at a round table. One of the chairs around this table is next to a window. If the diners are seated at random, what is the probability that the person paying the bill is seated next to the window?**

Since the people are seated around a table with a fixed reference point, this is a linear permutation. So there are $4!$ or 24 ways in which the people can be seated around the table. The number of favorable outcomes is the number of permutations of the other 3 diners given that the person paying the bill sits next to the window, $3!$ or 6.

So, the probability that the person paying the bill is seated next to the window is $\frac{6}{24}$ or $\frac{1}{4}$.

You Try: Suppose 7 points on a circle are chosen at random. Using the letters A through E, how many ways can the points on the circle be named?

Circular
(no fixed reference point)

$(7-1)! = 6! = 720$ possible ways